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This report summarizes algorithms developed for the solution and inversion of the Helmholtz and Maxwell Equations

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FOR MEDICAL IMAGING AND GEOPHYSICS

FINAL REPORT

AUTHOR OF REPORT: Frank Stenger

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a.2 "Rivlin's Theorem on Walsh Equiconvergence", (with A. Sharma, H.P. Dikshit, and V. Singh), J. Approx. Theory, 52 (1988) 339-349.

a.3. "Multigrid-Sinc Methods", with S. Schaffer, Applied Math. and Computation J. 19 (1986) 35-45.

a.4. "Inverse Scattering Solutions to the Exact Riccati Wave Equations by Iterative Rytov Approximations and Internal Field Calculations", with W.W. Kim, M.J. Berggren, S.A. Johnson, and C.H. Wilcox, in IEEE Ultrasonic Symposium (1985) 878-882.

a.5. "Performance of Fast Inverse Scattering Solutions for the Exact Helmholtz Equation Using Multiple Frequencies and Limited Views, with M.J. Berggren, S.A. Johnson, B.L. Carruth, W.W. Kim and P.L. Kuhn, Acoustical Imaging, v. 15 (1986) 193-201.

a.6. "Rational Approximation of the Heaviside and Delta Functions", with Y. Ikebe and M. Kowalski, submitted.

a.7. "Positive Operator Approximation on R^n", submitted.

a.8 "A Nonlinear Two-Phase Stefan Problem with Melting Point Gradient: a Constructive Approach", with D.D. Ang and Fritz Keinert, J. Comput. Appl. Math. 23 (1988) 245-255.

a.9. "Viscoplastic Flow Due to Penetration", with D.D. Ang, E.S. Folias and F. Keinert, to appear in Int. J. Fracture, (1988).

a.10. Approximate Methods in Control Theory, to appear in "Computational Methods in Control Theory", Ed. J. Lund, Elsevier (1989).

a.11. "A Sinc Algorithm for the Solution of Cauchy-Type Singular Integral Equations", with B. Bialecki, Math. Comp. 50 (1988) 761-783.

a.12. "Analysis of Inverse Scattering Solutions from Single-Frequency Transmission /data for the Helmholtz and Riccati Exact Wave Equation, (with M.J. Berggren, S.A. Johnson, W.W. Kim and C.H.Wilcox, Acoustical Imaging 15 (1987) 359-369.

a.13. "Optimal Complexity Recovery of Band and Energy Limited Signals", with M. Kowalski, J. on Complexity (1988), to appear.

WORK IN PROGRESS UNDER ARO SUPPORT

b.1. "Electromagnetic Scattering due to an Axially-Symmetric
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b.2. "A Poisson Solver In Two Dimensions", with R.B. Kearfott
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b.3. Sinc Package of Algorithms, with M. O'Reilly, in preparation.

b.4. An Explicit Procedure for the Linear Inversion of the
     Maxwell Equations, with G. Liao, in preparation.

b.5. Eigensolution of the Laplace-Beltrami Equation over a portion
     of the Sphere, with P. Li and A. Treibergs, in preparation.

b.6. Solution of an Inverse Stefan Problem, with D.D. Ang,
     in preparation.

SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED

STUDENTS WHO HAVE BEEN SUPPORTED BY THIS CONTRACT:

 B. Bialecki, completed Ph.D. in 1987.
 Ph.D. thesis topic:
a.14. "SINC Methods of Solving CSIE and Inverse Problems".

 M. Stromberg, completed Ph.D. in 1988.
 Ph.D. thesis topic:
a.15. "Solution of Shock Problems via Sinc Methods and Methods
       of Extrapolation to the Limit".

A previous relevant thesis, which is important to this project is
that of my student W. Faltenbacher. It is:

a.16. "A Numerical Integral Solution of a Scattering Problem on a
       Half Space", by Wolfgang Faltenbacher, which was completed
       in 1984.

        BODY OF THE REPORT

1. THE PROBLEM

The equations studied for purposes of inversion were

$$(1.1) \qquad u(\bar{r}) = u^{i}(\bar{r}) + k^2 \iiint_{V} G(\bar{r} - \bar{r}') \, f(\bar{r}') \, u(\bar{r}') \, d\bar{r}'$$

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$$(1.1) \qquad u(\bar{r}) = u^i(\bar{r}) + k^2 \iiint_V G(\bar{r} - \bar{r}') \, f(\bar{r}') \, u(\bar{r}') \, d\bar{r}'$$

and

$$(1.2) \quad E(\bar{r}) = E^{\iota}(\bar{r}) + k^2 \iiint_V [\; I - \nabla'\nabla' \; G(\bar{r} - \bar{r}') \;] \; f(\bar{r}') \; E(\bar{r}') \; d\bar{r}'$$

A rapidly convergent algorithm for solving the forward problem corresponding to (1.1) was carried out in a.16 above. Eq. (1.1) (rsp. (1.2)) obtains from the Helmholtz differential equation (rsp. Maxwell differential equations). Inversion involves the reconstruction of the complex scalar function $f(\bar{r})$ by applying inputs $u^{\iota}(\bar{r})$ (rsp. $E^{\iota}(\bar{r})$) and then measuring $u(\bar{r})$ (rsp. $E(\bar{r})$) on the boundary of V.

2. REMARK: In carrying out the solution to these problems, nearly all of our effort was directed towards the inversion of Eq. (1.1) for the following reasons:

(a) Equation (1.1) is a scalar equation, and hence the amount of work, the size of the computer program and storage, and the amount of time required to run a program are all considerably less than the corresponding ones for Eq. (1.2).

(b) Equation (1.2) has higher order singularities than Eq. (1.1), and the correct treatment of these requires additional computer code. In particular, it requires the evaluation of integrals of the type in (1.1) as well as the evaluation of principal value-type integrals. In the references [a.12,a.15] above, we developed the tools which are required to handle the principal value-type singularities.

(c) Except for the problem size and the evaluation of the principal value integrals, all other aspects of the inversion of (1.2) are the same as those for (1.1). That is, having developed an effective algorithm for the inversion of (1.1), we can, in effect, "write down" the corresponding one for the inversion of (1.2).

3. BRIEF SUMMARY OF RESULTS OBTAINED

During the duration of this proposal, we have developed some effective algorithms for inverting Eq. (1.1), which are described in the papers [a.5, a.16]. The other papers listed above, which describe work that was carried out with at least partial support of this proposal, involved the development of new mathematics and new algorithms that were required for the inversion of Eqs. (1.1) or (1.2).

4. MORE DETAILED SUMMARY OF RESULTS OBTAINED

We believe that we have made important advancements in developing new methods for the inversion of Eqs. (1.1) and (1.2). We now summarize some of these.

4.1 Evaluation and Inversion of the Laplace Transform.

This is the manuscript a.1. A copy of this manuscript was sent to your office earlier. We feel this is a very important paper, since it makes the application of Laplace transform inversion just as easy as the inversion of Fourier transforms. Method III in this paper is the simplest, and I expect that it will become a popular method for Laplace transform inversion. Both the evaluation and inversion of the Laplace transform are discussed in a.10.

4.2. Multigrid-Sinc Methods

The combination of multigrid and Sinc methods provides a natural "marriage", for solving both differential and integral equations, as was shown in a.3. We are in the process of developing an effective computer algorithm, which we expect will make both methods more popular.

4.3. Sinc Solution of Cauchy Singular Integral Equations.

The paper a.11 consists of yet another "breakthrough", since since it makes possible the very efficient (optimal) solution of a two-dimensional problem via a one dimensional procedure. It will thus make it possible to develop methods for solving a large class of three dimensional elliptic PDE problems via two dimensional methods.

4.4. Poisson Solver

We have done long and painstaking work on the paper b.2, which also represents a "breakthrough" similar, yet different from the one in 4.3 above. This, too, will open many new avenues for rapid solution of partial differential equations.

4.5. Sinc Package of Algorithms.

We expect completion of this work (b.3 above) during the next three months. It will appear in "Sinc Methods in Numerical Analysis", a textbook which is in the final stages of completion. Many of the procedures referred to above, as well as others which were previously supported by ORA will appear in this text.